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Electromagnetically induced transparency in a non-uniform plasma

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Abstract. Electromagnetically induced transparency in a non-uniform plasma is investigated. We discuss the possible frequency ranges in which a weak beam below cutoff can propagate transparently in this plasma with the presence of a strong high frequency field. We also analyze the effect of driving intensity on the frequency range of the probe.

PACS. 42.50.Hz Strong-field excitation of optical transitions in quantum systems; multi-photon processes; dynamic Stark shift – 52.35.Mw Nonlinear waves and nonlinear wave propagation (including parametric effects, mode coupling, ponderomotive effects, etc.)

1 Introduction

It is commonly believed that a weak probe beam will be strongly absorbed at its resonance frequency if most of the atoms are in the lower level. However, one can render a medium transparent to the weak probe radiation by the effect termed electromagnetically induced transparency (EIT) [1]. It is a technique where the complex refractive index of the probe beam (absorption and dispersion) can be modified by coupling additional coherent fields with other atomic transitions. EIT can be achieved by applying two lasers whose frequencies differ by a nonallowed transition of an atom as that in the first discussion [2]. Recently EIT in atoms has been studied much more widespreadly because of itself and its interesting application to laser without inversion [3] and nonlinear optics [4]. In the end of 1996, Harris reported [5] that a collective excitation of ideal uniform plasma rather than an internal excitation of an atom can also be used to establish transparency. He found that the role of the nonallowed transition of a single atom could be replaced by a collective longitudinal plasma oscillation. In order to realize and test Harris' results in a laboratory, a more realistic model has to be considered instead of the uniform one.

In this paper, we investigate EIT in a non-uniform plasma whose density varies along the laser beam propagation direction which is a simplification of a positive column of a glow discharge plasma. We analyze the experimental requirements for the laser frequencies and power density of driving laser theoretically. These analyses are useful and directive to an experimentalist in the field.

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2 Theory

By the application of the basic physics of EIT for a plasma system, Harris demonstrated that an electromagnetic wave with small amplitude can propagate below cutoff when a high frequency wave with large amplitude is presented in a uniform plasma [5]. The related energy scheme is shown in Figure 1. The frequency of the weak field, which is initially below cutoff and around which a passband is to be created, is denoted by ω_{s}. The frequency of the strong driving field, ω_{a}, is higher than the plasma frequency ω_{p}.

The propagation constant of the weak probe in the ideal plasma could be expressed as [5]:

$$K_{\mathrm{s}}(\delta\omega) = \frac{-\omega_{\mathrm{pole}} K_{\mathrm{a0}} \pm K_{\mathrm{s0}}[\delta\omega(\delta\omega - \omega_{\mathrm{crit}})]^{1/2}}{(\delta\omega - \omega_{\mathrm{crit}})} \tag{1}$$

$$\omega_{\mathrm{crit}} = \frac{{\omega_{\mathrm{a}}}^2 - {\omega_{\mathrm{s}}}^2}{{\omega_{\mathrm{p}}}^2 - {\omega_{\mathrm{s}}}^2} \omega_{\mathrm{pole}} \tag{2}$$

$$\omega_{\mathrm{pole}} = \frac{1}{2} \Lambda \omega_{\mathrm{p}} \tag{3}$$

where $\delta\omega = \omega_{\mathrm{p}} - (\omega_{\mathrm{a}} - \omega_{\mathrm{s}})$ is the frequency detuning of the plasma frequency from the longitudinal plasma oscillation frequency. $\Lambda = (q^2|E_{\mathrm{a}}|^2/4m\omega_{\mathrm{a}}^2)/mc^2$ is proportional to the power density of the driving laser field. The critical frequency, ω_{crit}, is the frequency where the refractive index changes from imaginary to real. From equation (1), it is clear that the transparent passband is from zero to the critical frequency ω_{crit}.

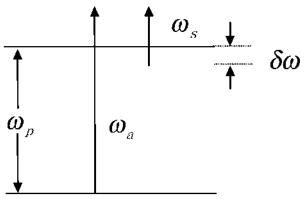

Fig. 1. Energy schematic for EIT in a uniform plasma. ω_a, ω_s are frequencies of driving and probe field, respectively. $\delta\omega = \omega_p - (\omega_a - \omega_s)$.

To extend the discussion from a uniform plasma to a non-uniform one, we have to think about how an experimental arrangement could be and what kinds of requirements for the lasers and the plasma are necessary. For this purpose, we consider a situation in which two lasers are collinearly and simultaneously incident perpendicularly to a cylindrical plasma column. Here, we simplify the characteristics of the plasma column so that it has a radial distribution of the plasma frequency (proportional to the root of plasma density) as shown in Figure 2. Also, we neglect the effect of the curvature of the column, resulting in the one-dimensional system along the Z-axis.

Unlike a uniform case where electron density n_e is uniform and the plasma frequency ω_p is a constant throughout the plasma, here the plasma frequency is linearly changed in both sides. The plasma frequency reaches its maximum value ω_{p0} between B and B′ where the electron density is uniform. We assume that the frequency of the driving field ω_a and the frequency of the probe field ω_s are in the optical region, and $\omega_s < \omega_{p0}$, $\omega_a > \omega_{p0}$. In this discussion, the density scale length of the plasma is much longer than the wavelength of each electromagnetic field, which is well satisfied for the optical range in the electromagnetic wave. The incident probe beam could go through the plasma from point O up to point A because $\omega_s > \omega_p$ in this segment. In the absence of the driving field, it will be reflected from A because the condition $\omega_s < \omega_{p0}$ is assumed. With the presence of driving field, a passband could be created and the probe wave could go through the non-uniform plasma transparently in a certain frequency range according to Harris' discussion. In order to get the frequency range, the probe frequency detuning must be within the passband with respect to every point in the segment from A to B, *i.e.*, if $\omega_s \le \omega_p \le \omega_{p0}$, then

$$0 \le \delta\omega \le \omega_{\mathrm{crit}}. \tag{4}$$

It means that at point A,

$$0 \le \omega_s - (\omega_a - \omega_s) \le \frac{\omega_a^2 - \omega_s^2}{\omega_{p0}^2 - \omega_s^2}\omega_{\mathrm{pole}} \tag{5}$$

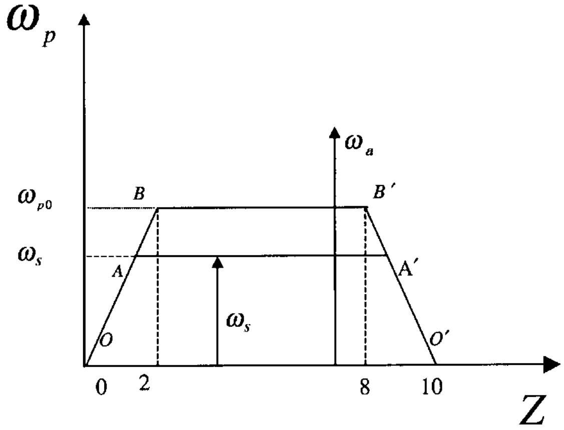

Fig. 2. The plasma frequency ω_p distribution along the propagation direction of the probe wave.

where ω_p in $\delta\omega$ and ω_{crit} has been replaced by ω_s. At point B equation (4) becomes:

$$0 \le \omega_{p0} - (\omega_a - \omega_s) \le \frac{\omega_a^2 - \omega_s^2}{\omega_{p0}^2 - \omega_s^2}\omega_{\mathrm{pole}} \tag{6}$$

where ω_p in $\delta\omega$ and ω_{crit} has been replaced by ω_{p0}. It is easy to understand that if equations (5, 6) are satisfied at the same time, equation (4) can be fulfilled at any point between A and B, therefore the probe beam could go through AB segment. Equations (5, 6) can be expressed in the forms below:

$$\omega_a \le 2\omega_s \tag{7}$$

and

$$\Lambda\omega_{p0}\omega_a^2 + (\omega_{p0}^2 - \omega_s^2)\omega_a - (\omega_s + \omega_{p0})(\omega_{p0}^2 - \omega_s^2) - \Lambda\omega_s^2\omega_{p0} \ge 0. \tag{8}$$

The solution to (7, 8) is

see equation (9) below.

The solution to (9) requires

see equation (10) below

which can be simplified as:

$$3\Lambda\omega_s^2\omega_{p0} + (\omega_{p0}^2 - \omega_s^2)(\omega_s - \omega_{p0}) \ge 0. \tag{11}$$

$$\frac{-(\omega_{p0}^2 - \omega_s^2) + \sqrt{(\omega_{p0}^2 - \omega_s^2)^2 + 4\Lambda\omega_{p0}[(\omega_s + \omega_{p0})(\omega_{p0}^2 - \omega_s^2) + \Lambda\omega_s^2\omega_{p0}]}}{2\Lambda\omega_{p0}} \le \omega_a \le 2\omega_s \tag{9}$$

$$2\omega_s \ge \frac{-(\omega_{p0}^2 - \omega_s^2) + \sqrt{(\omega_{p0}^2 - \omega_s^2)^2 + 4\Lambda\omega_{p0}[(\omega_s + \omega_{p0})(\omega_{p0}^2 - \omega_s^2) + \Lambda\omega_s^2\omega_{p0}]}}{2\Lambda\omega_{p0}} \tag{10}$$

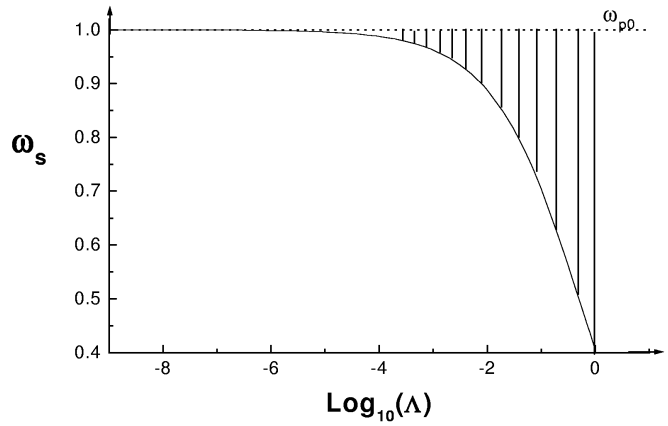

Fig. 3. The transparent range of probe frequency *versus* Λ.

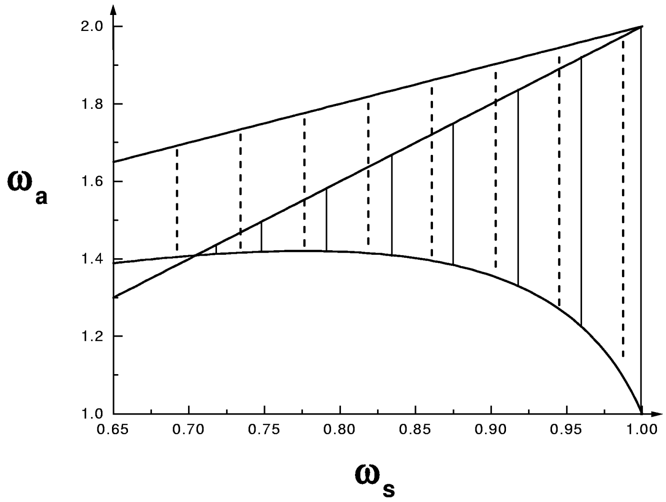

Fig. 4. The range of driving field frequency *versus* the probe frequency. The area depicted by the dotted lines is for uniform plasma and the solid lines is for non-uniform one. The parameters used are $\omega_{p0} = 1.0$, $\Lambda = 0.1$.

Equation (11) gives the possible range of probe frequency in which EIT can be produced in the positive column plasma. It is clear that the frequency range depends on the power density of the driving laser. This means that the probe frequency ω_s is limited in this plasma, while it could be any value from zero to ω_p in a uniform plasma.

On the other hand, the range of driving frequency ω_a for EIT in the positive column plasma can be obtained by solving equation (9) under the condition that ω_s satisfies equation (11). The range of ω_a in the non-uniform plasma is just one part of the corresponding range in a uniform plasma.

3 Results and discussion

This section is organized as two parts. In the first part, we discuss what parameters could be for the realization of the experiment. In the second part, we test the theoretical analysis in Section 2 by the normalized parameters.

(1) We focus on what needs for in the experiment where a driving field controls the absorption of the probe beam in a plasma which can be simplified as the model shown in Figure 2. We can choose the frequencies of the two fields at $10^9 \sim 10^{14}$ Hz, and so the plasma frequency. That means the plasma density at $10^{16} \sim 10^{26}$ m^{-3}. The requirement for the power density of the driving field can be estimated from the parameter Λ. For example, when $\Lambda = 0.1$, the power density for the driving field is equal to $10^4 \sim 10^{14}$ W/cm^2 by the corresponding frequency of the driving field at $10^9 \sim 10^{14}$ Hz.

(2) In order to examine the physics basis for EIT in the plasma, we deal with the theoretical analyses using specific parameters.

First, we estimate the effect of power density of the driving laser on the transparent frequency window of the probe from equation (11). The result is shown in Figure 3. The frequency of the probe field ω_s for EIT is indicated in the shaded area of Figure 3. In the positive column plasma the possible range of ω_s for transparency is limited by the power density of the driving field, while the range of ω_s in a uniform plasma is not.

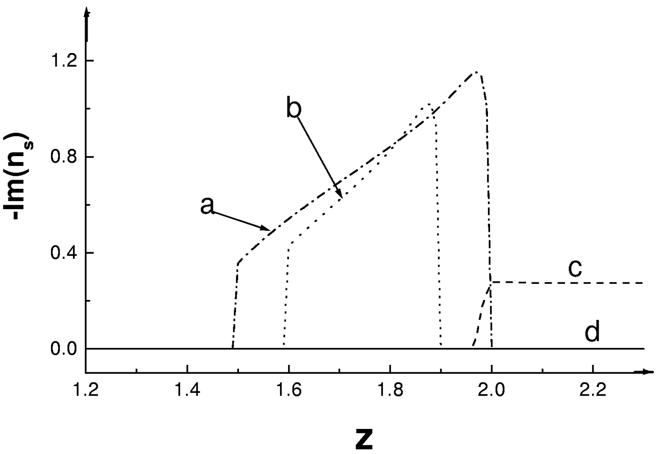

Fig. 5. The imaginary part of the refractive index of probe field $-\mathrm{Im}(ns)$ (proportional to probe field absorption) *versus* the distance in plasma. The parameters used are $\Lambda = 0.01$, $\omega_{p0} = 1.0$, $\omega_a = 1,75$, ω_s= 0.75, 0.80, 0.88, 0.95 for curves a, b, c, d, respectively.

According to equation (9), the allowed frequency range of the driving field ω_a can be found for a possible probe frequency ω_s when Λ and the plasma frequency ω_{p0} are fixed. The range is depicted by the solid lines in Figure 4. We can see that the frequency range of driving field becomes larger and larger when the probe frequency approaches ω_{p0}. The range of driving laser frequency for EIT under the same condition for a uniform plasma is depicted in the same figure by the dotted lines.

According to equation (1), the imaginary part of the refractive index of probe field, which is proportional to the absorption, is shown in Figure 5. The curves are drawn along the propagation direction on the left section of the plasma. The right part of the curve is symmetric with the left one (not shown). The parameters used in Figure 5 are the same as in Figures 3 and 4. In Figure 5, curves a and b show the situations where frequency ω_s is out of the shaded area in Figure 3, *i.e.*, equation (11) is not satisfied. Curve c shows the situation that equation (11) is satisfied but equation (9) is not. Curve d shows the situation where both equation (9) and equation (11) are satisfied. We notice that EIT can not be realized unless equation (9) and equation (11) are satisfied at the same time.

4 Conclusion

In this paper, we adopt a cylindrical column plasma to extend the discussion of EIT from a uniform plasma to a non-uniform one. We find that the transparent range of probe frequency is limited by the power density of driving field, and the range of driving field frequency ω_a is smaller in the non-uniform plasma than that in the uniform one.

We should stress that our calculations are based on the results of Harris. We discuss the frequency conditions for EIT in a non-uniform plasma only. The actual form of the density profile is not relevant to most of the analysis. Indeed the conditions (7–9) and Figures 3 and 4 do not depend on $\omega_p(z)$, but only on the maximum value of ω_p, namely ω_{p0}. The profile of $\omega_p(z)$ only influences the form of the curves of Figure 5. Our results show that the frequency ranges for EIT in this model are narrower than that in the uniform one. These restrictions must be taken into account in an experimental test.

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